FM
Frequency Modulation

Let $m(t)$: message signal at baseband ($f\downarrow$)
$c(t)$ Carrier signal at passband ($f\uparrow$)

where:-

$$C(t) = A_c \cdot \cos(\theta_i(t))$$
$$c(t) = A_c \cos(2\pi f_c \cdot t)$$

$\boxed{A_c}$ Carrier Amplitude
if $m(t)$ modulates the amplitude → AM

$\boxed{\theta_i(t)}$ Carrier angle
if $m(t)$ modulates the angle → Angle Modulation

• **Angle Modulation:-**

In angle modulation, the phase angle of the carrier $\theta_i(t)$ is varied according to the message signal $m(t)$. not the amplitude as in AM.

Angle modulation

① **Phase modulation (PM)**

تتغير $\theta_i(t)$ بشكل مباشر مع ال $m(t)$.

$$\theta_{i_{Pm}}(t) = 2\pi f_c t + k_p \cdot m(t)$$

k_p: phase modulation Sensitivity

PM equation
$$S_{PM}(t) = A_c \cos(2\pi f_c t + k_p \cdot m(t))$$
Carrier after modulation

② Frequency Modulation FM

حيث يتغير تردد ال $c(t)$ بشكل مباشر مع ال $m(t)$

$$f_c' = f_c + K_f \cdot m(t)$$

↳ التردد بعد التعديل

$$\because \theta = \int \omega \cdot dt$$

$$\theta_i(t) = 2\pi \int f_c' \, dt = 2\pi f_c t + 2\pi K_f \int m(t)\, dt$$

$$\boxed{S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int m(t)\, dt\right)}$$

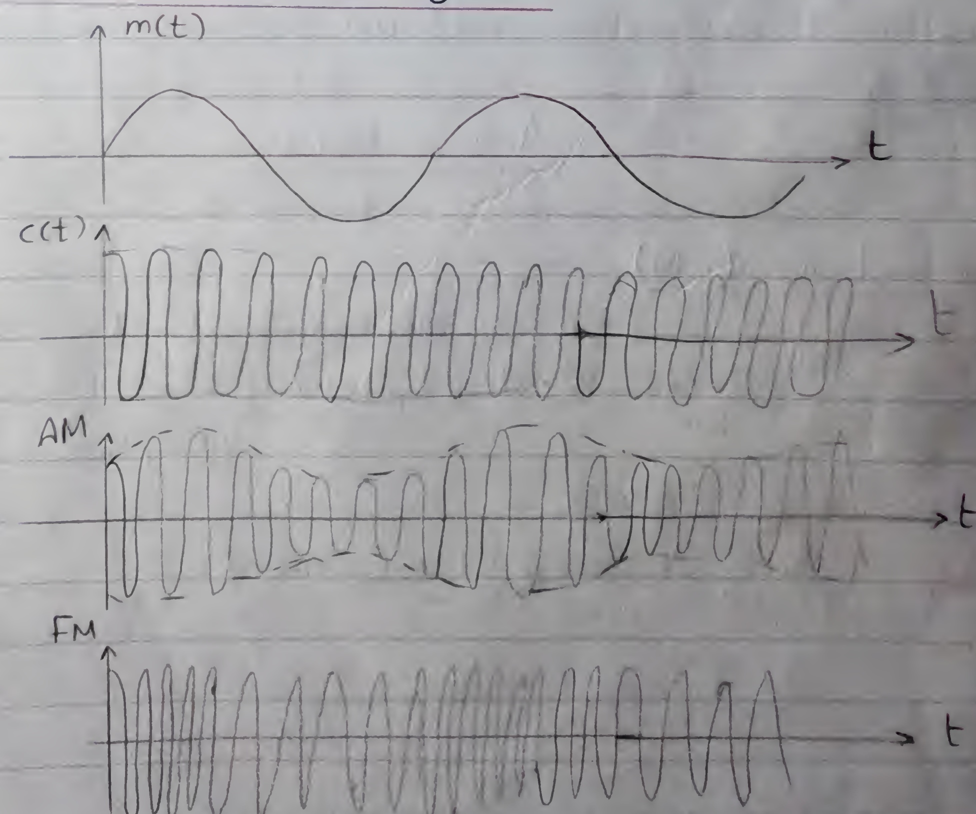
→ fM equation

$$f_i(t) = f_c + K_f \cdot m(t)$$

↳ instantonous frequency

K_f: Frequency modulation Sensitivity

Time Representation of modulated Signals :-



عند مقارنة الـ FM بالـ PM نلاحظ أنه يمكن توليد الـ FM من الـ PM والعكس صحيح .

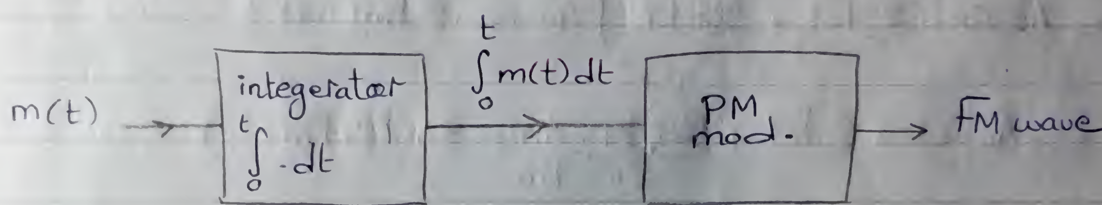
Q: How to generate FM from PM & PM from FM ??

Sol.

$$\because S(t)_{PM} = A_c \cos(2\pi f_c t + k_p m(t))$$

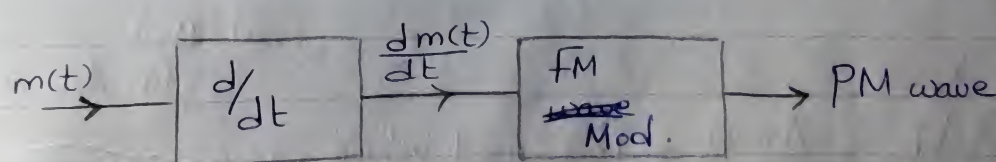
$$S(t)_{FM} = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt)$$

ⓐ FM from PM



وضعنا integerator بعد m(t) لكي نحصل على $\int_0^t m(t)$

ⓑ PM from FM



وضعنا d/dt بعد الـ m(t) لكي نحصل على تفاضل الـ m(t) كما بالرسم .

Single Tone FM modulation

① Narrow Band FM
N.B.F.M

② Wide Band FM
W.B.F.M

let $m(t) = A_m \cos 2\pi f_m t$, B.W of $m(t) = f_m$

$c(t) = A_c \cdot \cos 2\pi f_c t$, $f_c \gg f_m$

$$S_{fm}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t)\,dt)$$

$$S_{fm}(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t)\,dt)$$

$$= A_c \cos(2\pi f_c t + \frac{\cancel{2\pi} k_f \cdot A_m}{\cancel{2\pi} \cdot f_m} \sin(2\pi f_m t))$$

let $\Delta f = k_f \cdot A_m$ max. freq. deviation

حفظ

أقصى انحراف عن f_c

$\because f_c' = f_c + \underbrace{k_f\, m(t)}$

الانحراف عن f_c وله أقصى قيمة $k_f \cdot A_m$

حفظ

let $\dfrac{\Delta f}{f_m} = \dfrac{A_m \cdot k_f}{f_m} = \beta$ modulation idex

بالتعويض عن β في المعادلة الأخيرة

حفظ وهام جداً

$$S_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

معادلة الـ FM العامة

$\beta = \frac{\Delta f}{f_m}$ modulation index

if $0 < \beta < 1$
$\therefore$ N.B.F.M

if $1 \leqslant \beta$
$\therefore$ W.B.F.M

ⓐ For N.B.F.M

$$S(t)_{FM} = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t)) \quad , \quad 0 < \beta < 1$$

Note that:- $\cos(A+B) = \cos A \cos B - \sin A \sin B$

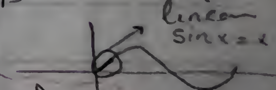
$$S(t)_{FM} = A_c \cos(2\pi f_c t) \cdot \cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t)).$$

since $0 < \beta < 1$ $\quad$ β very small

$$\cos(\beta \sin(2\pi f_m t)) \simeq \cos 0 \simeq 1$$
$$\sin(\beta \sin(2\pi f_m t)) \simeq \beta \sin(2\pi f_m t)$$

$\cos(v.small) \simeq 1$

for small x
linear
$\sin x \simeq x$



$$\therefore \underset{N.B.F.M}{S(t)} = \underbrace{A_c \cos(2\pi f_c t)}_{c(t)} - \beta (A_c \sin(2\pi f_c t)) \cdot (\sin(2\pi f_m t))$$

الصيغة النهائية لل N.B.F.M

Spectrum of NBFM

$$\underset{NBFM}{S(t)} = \underbrace{A_c \cos(2\pi f_c t)}_{c(t)} - \underbrace{\beta (A_c \sin(2\pi f_c t))(\sin(2\pi f_m t))}_{D.S.B}$$

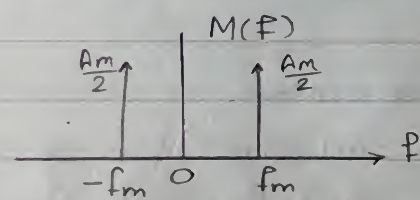
$$= A_c \cos(2\pi f_c t) - \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m) t + \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m) t$$

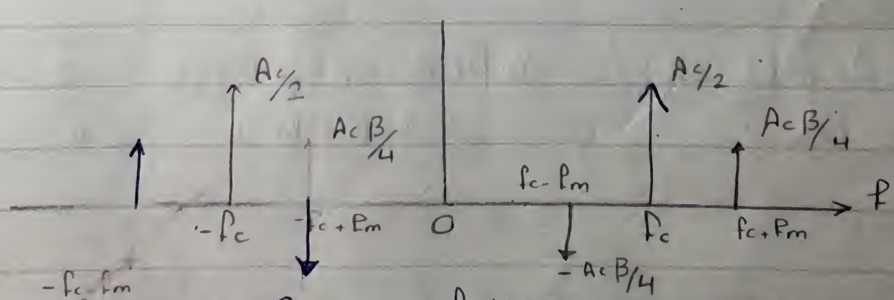
$$= \underset{c(t)}{\underset{\downarrow}{A_c \cos(2\pi f_c t)}} + \underset{U.S.B.}{\underset{\downarrow}{\frac{\beta A_c}{2} \cos(2\pi (f_c + f_m) t)}} - \underset{L.S.B}{\underset{\downarrow}{\frac{\beta A_c}{2} \cos(2\pi (f_c - f_m) t)}}$$

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{\beta A_c}{4} [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]$$

$$- \frac{\beta A_c}{4} [\delta(f + (f_c - f_m)) + \delta(f - (f_c - f_m))].$$

- الشكل التالى يوضح ال NBFM Spectrum





Spectrum of NBFM

هام جداً جداً

NBFM Power

P_{total} = total transmitted Power = P_t

$$P_t = P_c + P_{U.S.B} + P_{L.S.B}$$

* $P_c = \frac{A_c^2}{2}$ * $P_{L.S.B} = P_{U.S.B} = \frac{A_c^2 \beta^2}{8}$

* $P_{D.S.B} = \frac{A_c^2 \beta^2}{4}$

* $P_t = P_c + P_{DSB} = P_c\left(1 + \frac{\beta^2}{2}\right)$

$$\boxed{\begin{aligned} &B.W. \text{ of } m(t) = f_m \\ &B.W. \text{ of NBFM} = 2f_m \end{aligned}}$$

حفظ

نلاحظ أنه الـ NBFM يشبه الـ AM ولكن أيهما أفضل ؟؟

بالرغم من أن الـ NBFM يشبه الـ AM في الباور والـ BW والـ Spectrum إلا أنه الـ NBFM أفضل لأن الـ NBFM له مقاومة عالية للـ noise ولذلك الـ NBFM على الـ AM .

(b) for W.B.F.M

$\beta \geq 1 \longrightarrow$ بالتالي لن نستطيع إختصار $\cos(\beta \sin(\,))$ و $\sin(\beta \sin(\,))$

كما في ال Narrow Band فهتختلف المعادلة هنا

$$S(t)_{WBFM} = \sum_{n=-\infty}^{\infty} \underbrace{J_n(\beta)}_{\text{Bessel's } f_n} A_c \cos(2\pi (f_c + n f_m)t)$$ مهم

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c + n f_m)) + \delta(f + (f_c + n f_m)) \right]$$ مهم

- $B.W.$ (Carson's Rule) $= 2(\beta + 1) f_m$ مهم $= 2(\Delta f + f_m)$

- $P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)$. which is $\frac{V_P^2}{2}$ of the Cosine signal

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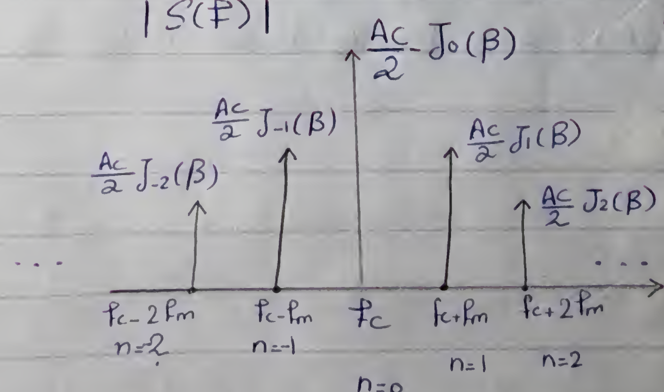
* Note :-

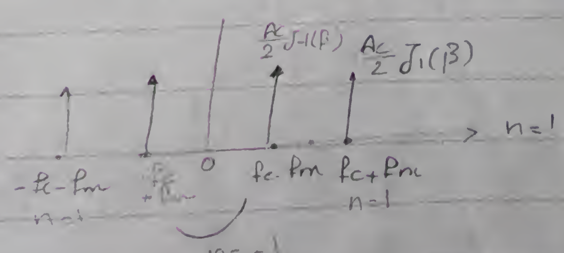


at $n=1$

$\frac{A_c}{2} J_1(\beta) [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]$

$n=-1$.

$\frac{A_c}{2} J_{-1}(\beta) \cdot [\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))]$



B.W is extended to infinity (Wide Band) but Carson's rule only consider $B.W = 2(\beta + 1) f_m$ and as it shows the high components has low amplitude that can be ignored.

In FM Questions

① Find the modulation index (β)

Given f_m , Δf

$$\beta = \frac{\Delta f}{f_m}$$

Given: الصورة العامة لمعادلة f_m
و استخرج منها ال β

② NBFM or WBFM ??

$0 < \beta < 1$
NBFM
لو طلب
• $B.W. = 2 f_m$

• $$\text{Power} = P_c + P_{DSB} = \frac{A_c^2}{2} + \frac{A_c^2 \beta^2}{4} = \frac{A_c^2}{2}\left(1 + \frac{\beta^2}{2}\right)$$

$$P_T = P_c\left(1 + \frac{\beta^2}{2}\right)$$

نفس الباور لل DSBTC بس
بدل ال $\mu \leftarrow \beta$

$\beta \geq 1$
WBFM
لو طلب
• Carson's rule
$B.W. = 2(\beta + 1) \cdot f_m$

• $$\text{Power} = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$
نعرفها من جدول Bessel

لو طلب ال Spectrum
عدد ال sidebands حول ال f_c

$$n = \frac{B.W}{2 f_m}$$

ونحسب الباور على هذا الجزء فقط

* في الحالتين التردد اللحظي $f_i(t) = f_c + k_f \cdot m(t)$ *

FM Sheet

Before answering any fM question determine from β is it a NBfM or WBfM ?!

① $\Delta f = 8\ \text{kHz}$ $\quad f_m = 4\ \text{kHz}$ $\quad$ BW of fm = ??

$A_c = 18\ \text{V}$ $\quad f_c = 3\ \text{MHz}$ $\quad$ FM eqn = ??

B.W. of fm

NBFM	WBFM
$2f_m$	$2(\beta+1)f_m$
$0<\beta<1$	$\beta \geq 1$

- $\beta = \dfrac{\Delta f}{f_m} = \dfrac{8}{4} = 2 > 1 \longrightarrow$ WBFM

- BW of FM Signal $= 2(\beta+1)f_m$
  $= 2(2+1)(4\ \text{KHz})$
  $= 24\ \text{KHz}$

$S(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$

- $S(t) = 18 \cos(2\pi \cdot 3 * 10^6 t + 2 \sin(2\pi \cdot 4 * 10^3 t))$

Note

$J_{-n}(\beta) = J_n(\beta)$ $\quad$ n even

$J_{-n}(\beta) = -J_n(\beta)$ $\quad$ n odd

$J_{-n}(\beta) = (-1)^n \cdot J_n(\beta)$

$f_i(t) = f_c + K_f \cdot A_m$

Hz   Hz $\frac{Hz}{V}$ V

② $A_m = 4V$ $f_m = 1200$ Hz $\omega = 2\pi f$

$A_c = 8V$ $f_c = 4$ MHz   rad/s   Hz or S^{-1}

$K_f = 5652$ rad/s/volt $\div 2\pi = 899.54$ Hz/s

$S(t) = ?$ $\Delta f = ?$ B.W. = ?

Sol.

$\beta = \frac{\Delta f}{f_m}$, • $\Delta f = K_f \cdot A_m = 899.5 * 4 = 3598$ Hz

$\beta = \frac{899.5*4}{1200} \cong 3 > 1 \longrightarrow$ ∴ WBFM

• $B.W. = 2(\beta+1) f_m$
$= 2(3+1)(1200)$
$= 9600$ KHz

$S(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$
$= 8 \cos(2\pi * 4*10^6 t + 3 \sin(2\pi * 1200 t))$

③ $S(t) = 10 \cos(2\pi(4*10^6)t + 0.8 \sin(2\pi \cdot 600 t))$ volts
$A_m = 4$ V

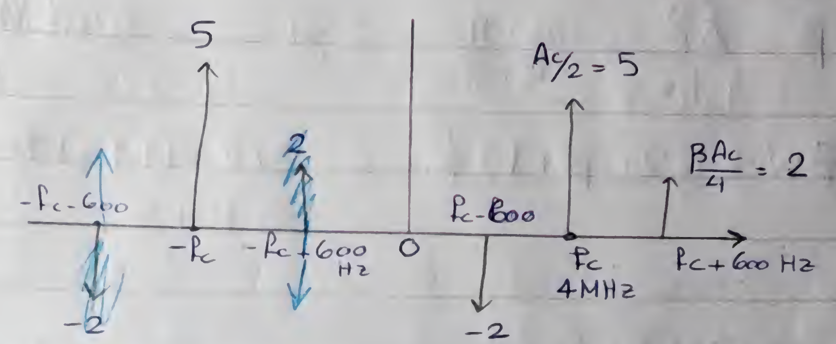
a) Modulation Type = ? B.W. = ? $P_e = ?$

$S(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$
∴ $A_c = 10$ V , $f_c = 4*10^6$ Hz, $f_m = 600$ Hz
$\beta = 0.8 < 1 \longrightarrow$ NBFM

$B.W = 2 f_m = 2 * 600 = 1200$ Hz $= 1.2$ KHz

$$P_c = \frac{A_c^2}{2} = \frac{10^2}{2} = 50 \text{ watts}$$

b) Spectrum = ? $\Delta f = ?$



$$\beta = \frac{\Delta f}{f_m} \longrightarrow \Delta f = \beta \cdot f_m = 0.8 * 600 = 480 \text{ Hz}$$

if asked about k_f : $k_f = \frac{\Delta f}{A_m} = \frac{480}{4} = 120$ Hz/volt

c) A_m 4V $f_m = 600$ $\xrightarrow{\text{Changed}}$ A_m' 7V $f_m' = 350$

$$\beta' = \frac{k_f \cdot A_m'}{f_m'} = \frac{120 * 7}{350} = 2.4 > 1 \longrightarrow \text{WBFM}$$

إتحول النظام
لـ WBFM بالتالي الـ B.W زاد

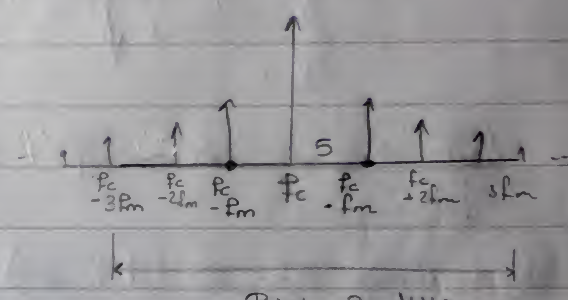
$$BW' = 2(\beta'+1) f_m'$$
$$= 2(1.4+1)(350)$$
$$= 1.68 \text{ KHz}.$$

(4) $\Delta f = 10\ KHz$ $f_m = 5\ KHz$

$A_c = 10\ V$ $f_c = 500\ KHz$

$$\beta = \frac{\Delta f}{f_m} = \frac{10\ KHz}{5\ KHz} = 2 > 1 \longrightarrow WBFM$$

$$B.W. = 2(\beta+1) f_m = 2(2+1)(5\ KHz)$$
$$= 30\ KHz$$

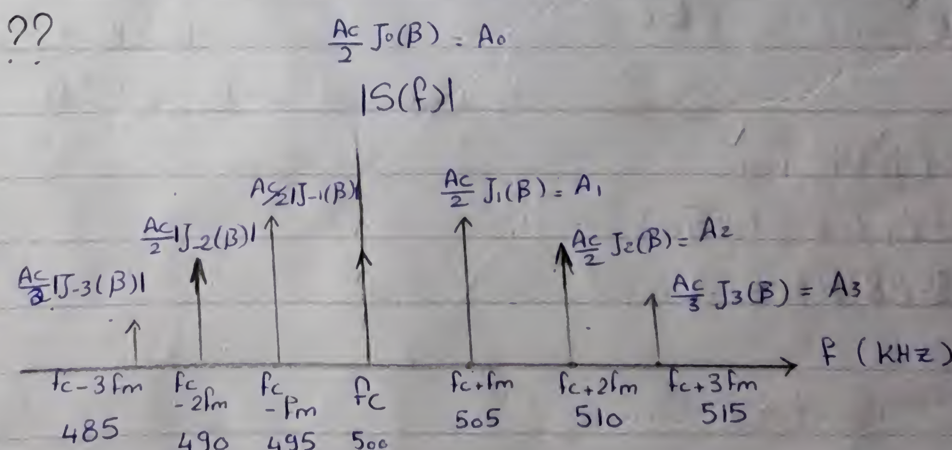


a) No. of sets of side frequencies $= \dfrac{30\ KHz}{2\ f_m}$

$$\frac{B.W}{2 f_m} = 3$$

$$\boxed{n=3}$$

b) Spectrum ??



$$A_0 = \frac{A_c}{2} J_0(2) = 5 * 0.2239 = 1.1195$$

$A_1 = 5 * J_1(2) = 5 * 0.5767 = 2.8835$

$A_2 = 5 * J_2(2) = 5 * 0.3528 = 1.764$

$A_3 = 5 * J_3(2) = 5 * 0.1289 = 0.6445.$

c) Power

$$P = \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta) = \frac{A_c^2}{2}\left[J_0^2(\beta) + J_1^2(\beta) + J_{-1}^2(\beta) + \cdots\right]$$

$$= \frac{A_c^2}{2}\left[J_0^2[\beta] + 2J_1^2(\beta) + 2J_2^2(\beta) + 2J_3^2(\beta)\right]$$

$$= 2\sum_{n=-\infty}^{\infty} A_i^2 = 2(A_0^2 + 2A_1^2 + 2A_2^2 + 2A_3^2) + \cdots$$

$$= 2A_0^2 + 4A_1^2 + 4A_2^2 + 4A_3^2 \quad \text{until } A_n \ (n=3 \text{ here})$$

$$= 2(1.1195^2 + 2 * 2.8835^2 + 2 * 1.764^2 + 2 * 0.6445^2)$$

$$= 2(1.1195)^2 + 4(2.8835)^2 \qquad A_{-1} = -A_1$$

$$+ 4(1.764)^2 + 4(0.6445)^2 \qquad \therefore A_{-1}^2 = A_1^2$$

$$= 49.87 \text{ watts}$$

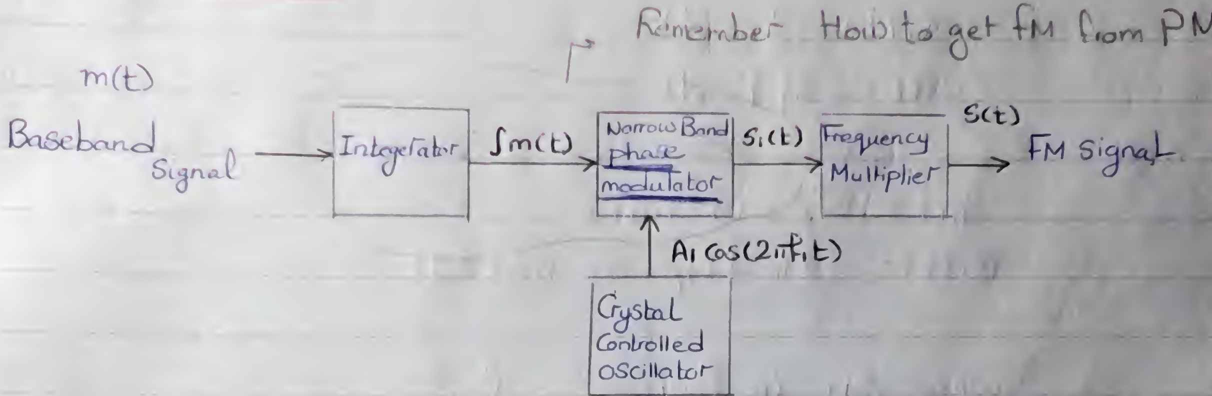
$\frac{A_c^2}{2} J_1^2(\beta) \quad \frac{A_c^2}{2} J_1(\beta)$

A^2

Generation of FM Waves :-

- There are two methods of FM waves generation :
  1. Indirect Method
  2. Direct Method

① Indirect FM



This modulator Produces WBFM signal from NBFM signal that's why it is called indirect.

$$S_1(t) \longrightarrow \text{NBFM}$$

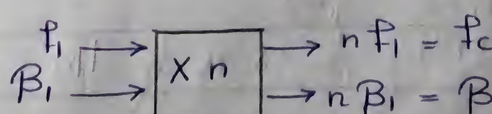
It is then multiplied in Frequency by factor n to Produce the desired WBFM.

$$S_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi K_f \int m(t)\,dt\right)$$

$$S_1(t) = A_1 \cos\left(2\pi f_1 t + \underbrace{\beta_1}_{\frac{k_f \cdot A_m}{f_m} = \frac{\Delta f}{f_m}} \sin(2\pi f_m t)\right)$$

For $m(t)$ cosine signal

- β_1 is Kept Small $\simeq 0.5$ to keep the distortion minimum in the medulator



$$S_1(t) = A_1 \cos\left(2\pi n f_1 t + 2\pi n K_f \int m(t)\,dt\right)$$

$$= A_c \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right).$$

Date

where $f_c = n f_1$
$\beta = n \beta_1$

So, by Properly choosing the multiple n, we can determine the modulation index β and f_c as desired.

توضيح :-

$$\theta_i(t) = \int \omega \, dt$$
$$\theta_i(t) = 2\pi \int f \cdot dt$$

After mod.

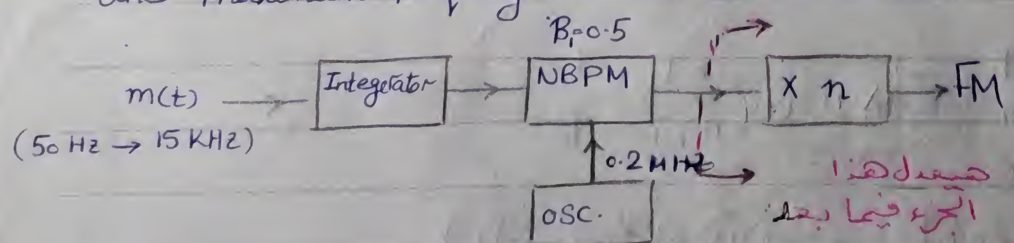
$$f_i(t) = f_1 + K_f \cdot m(t)$$
$$\theta_i(t) = 2\pi \int (f_1 + K_f \cdot m(t)) dt$$

when $\times n$ in freq. f

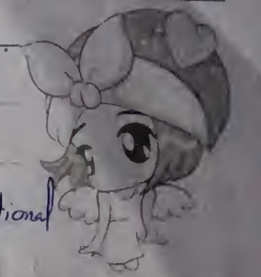
$$= 2\pi \cdot n \cdot \int \underbrace{f_1 + K_f \cdot m(t)} \, dt$$
$$\therefore \theta_i(t) = 2\pi \boxed{n} f_1 t + 2\pi \boxed{n} K_f \int m(t) \, dt$$

- This method can be unpractical & need more enhancing in Practical Systems ...

For example, it is required to send FM wave that has $f_c = 90$ MHz and maximum frequency deviation $\Delta f = 75$ KHz.



- the maximum β for NBPM to operate satisfactory is 0.5, the m(t) signal



has frequencies from 50 Hz to 15 KHz.

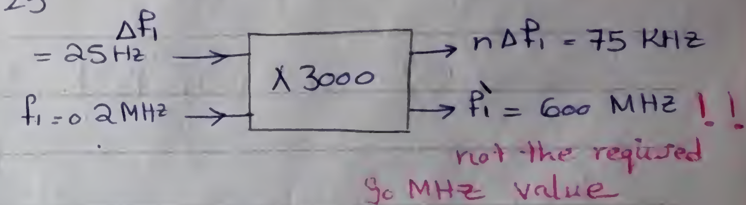
So, for least f_m, β must be 0.5 as β is inversely proportional to f_m.

$$\beta = \frac{\Delta f}{f_m}$$

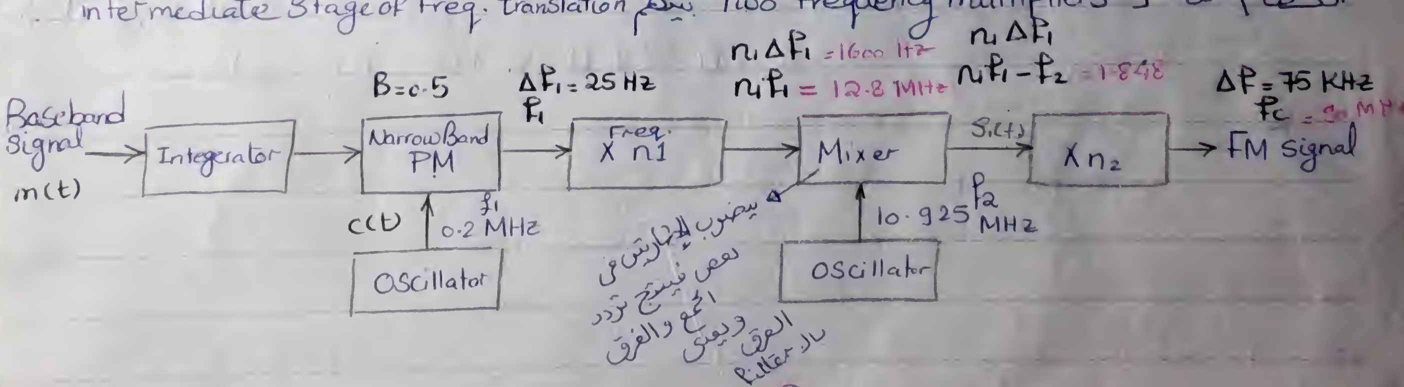
$$0.5 = \frac{\Delta f_1}{50} \longrightarrow \Delta f_1 = 0.5 \times 50 = 25 \text{ Hz}$$

Therefore, for maximum $\Delta f = 75$ KHz

$$n = \frac{75 \text{ KHz}}{25} = 3000$$



• يبقى الهدف دلوقت إني أقدر أحقق المواصفات كلها Δf وكمان f_c لذلك عدل النظام ده لـ Two frequency multipliers بينهم intermediate stage of freq. translation.



$$n_1 n_2 = 3000 \rightarrow (1)$$

$$\text{So, } \Delta f_1 = 25 \text{ Hz} \longrightarrow \Delta f = 75 \text{ KHz}$$

$$n_1 f_1 - f_2 = \frac{f_c}{n_2}$$

$$0.2 n_1 - 10.925 = \frac{90}{n_2} \rightarrow (2)$$

$$n_1 = 64.3 \simeq 64$$
$$n_2 = 46.7 \simeq 48$$

64 & 48 can be factorized by multiples of 2 & 3 which means that multiplication process can be done by X 2 and X3 freq multipliers.

3	48
2	16
2	8
2	4
2	2
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

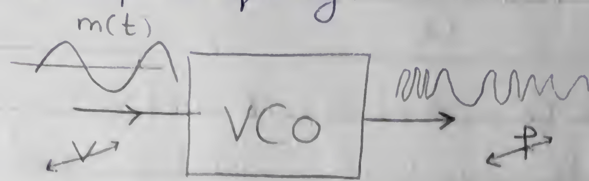
$$S_1(t) = A_1 \cos(2\pi f_1 t + B_1 \sin(2\pi f_m t)) * \cos(2\pi f_2 t)$$

$$= \frac{A_1}{2}\left[\cos(2\pi(nf_1 - f_2)t + \underbrace{B_1}_{\text{زي ما هي}} \sin(2\pi f_m t)\right] + \cos(\text{ المجموع })]$$

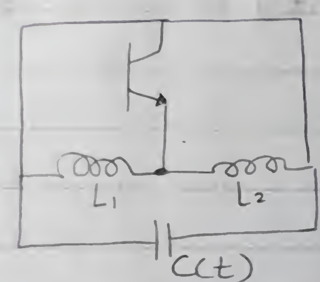
I   * لو في مسألة أداك معطى f_1, f_2 , f_c , Δf وطلب تعمل design يبقى
مطلوب n_1 و n_2 من المعادلة ① و ②

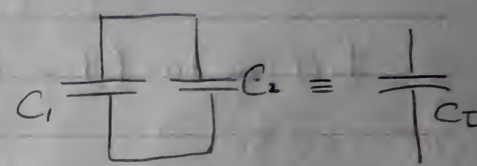
② Direct FM

The idea of VCO "Voltage-Controlled Oscillator" which its input voltage changes the o/p frequency.



One way to implement the VCO is by Hartley oscillator which has fixed Capacitor shunted by variable Capacitor which its capacitance changes with i/p Voltage.





$C_\tau = C_1 + C_2$

C(t): Total Capacitance (Fixed + Variable)
↳ changes with o/p voltage

The freq. of oscillation of Hartley

عوض

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,C(t)}}$$

التردد عند لحظة معتمد على C عند هذه اللحظة

$$C(t) = C_o + \Delta C \cos(2\pi f_m t)$$

و C(t) عند لحظة تعتمد على الـ m(t) عند هذه اللحظة

المتغير

↳ assume $m(t) \rightarrow \cos(2\pi f_m t)$

max. change at the variable C when m(t) = max.

$$f_i(t) = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{(L_1+L_2)C_o + (L_1+L_2)\Delta C \cos(2\pi f_m t)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1+\frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)}}$$

$$f_i(t) = f_o \cdot \left(1 + \frac{\Delta C}{C_0}\cos(2\pi f_m t)\right)^{-1/2}$$

$f_o \rightarrow$ freq at $C(t) = C_0$ which means $m(t)=0$.
↳ unmodulated freq. "without modulating signal $m(t)$".

$$(1+x)^n \simeq 1 + nx + \underbrace{\frac{n(n-1)}{2!}x^2 + \dots}$$

↳ v.small terms at ΔC small so, ignore them here

$$f_i(t) \simeq f_o \cdot \left(1 - \frac{\Delta C}{2C_0}\cos(2\pi f_m t)\right)$$

$\Delta C \longrightarrow$ max. value of var. C which is max. deviation from C_0
$C_0 \longrightarrow$ Causes f_o "$m(t)=0$"

$$\therefore \frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_o}$$

f_c $\qquad K_f \cdot m(t) = \underbrace{K_f \cdot A_m}_{\Delta f}\cos(2\pi f_m t)$

$$f_i(t) \simeq f_o + \Delta f \cos(2\pi f_m t)$$

which is the instantenous freq. of an FM wave.

$$\Delta f = \frac{1}{2\pi\sqrt{L_T \cdot \Delta C}} \qquad \sqrt{\frac{\Delta C}{C_0}}^{-1} = \frac{\Delta f}{f_o}$$

$$f_o = \frac{1}{2\pi\sqrt{L_T \cdot C_0}} \qquad 1 - \frac{\Delta C}{2C_0} = \frac{\Delta f}{f_o}$$

Demodulation of FM waves :-

- The Process that enables us to recover the original modulating wave from the frequency modulated wave.

- There are two methods for FM demodulation:
  1. Frequency discriminator
  2. Phase-Locked Loop.

- In both cases, the objective is to Produce a transfer charachteristic which is the inverse of that of the frequency modulator.

1 Frequency Discriminator

- It Consists of a slope circuit followed by an envelope detector.
- An ideal slope circuit is charachterized by a transfer function that is Purely imaginary, varying Linearly with the frequency inside the Prescribed frequency interval.

$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + \frac{B_T}{2}\right) & f_c - \frac{B_T}{2} \leq f \leq f_c + \frac{B_T}{2} \quad \rightarrow (1) \\ j2\pi a\left(f + f_c - \frac{B_T}{2}\right) & -f_c - \frac{B_T}{2} < f < -f_c + \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$= \tilde{H}_1(f - f_c)$

$f = f + f_c$

Transfer Fn. of slope circuit

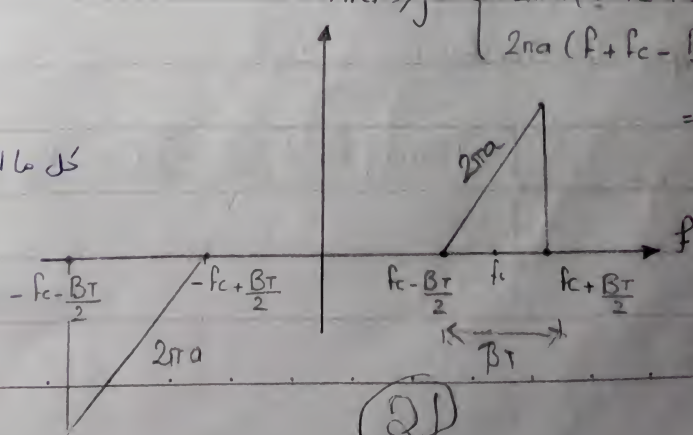
to get $\tilde{H}_1(f)$

$$y = mx + c = \underbrace{2\pi a}_{m} f - 2\pi a\left(f_c + \frac{B_T}{2}\right)$$

$$H_1(f)/j = \begin{cases} 2\pi a\left(f - f_c + \frac{B_T}{2}\right) & f_c - \frac{B_T}{2} \leq f \leq f_c + \frac{B_T}{2} \\ 2\pi a\left(f + f_c - \frac{B_T}{2}\right) & -f_c - \frac{B_T}{2} \leq f \leq -f_c + \frac{B_T}{2} \end{cases}$$

$$= 2\pi a f + 2\pi a\left(f_c - \frac{B_T}{2}\right)$$

كل ما ال f تزيد الجهد بيزيد



$$S(t) \xrightarrow[\text{FM wave}]{} \boxed{\text{Slope cct.}}^{\,H_1(f)} \longrightarrow S_1(t)$$

$B.W. = B_T$

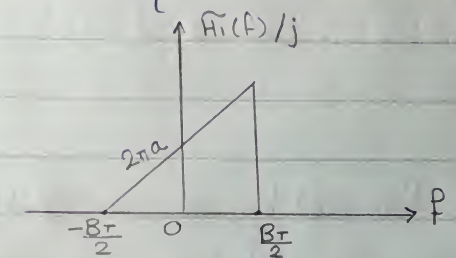
f_c

$\tilde{H}_1(f)$: Complex transfer fn. of slope circuit

$$\tilde{H}_1(f-f_c) = H_1(f) \quad f>0$$

∴ From ①

$$\tilde{H}_1(f) = \begin{cases} j\,2\pi a\left(f + \frac{B_T}{2}\right) & -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$



$S(t)$: the incoming FM wave

$$S(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$$

$\tilde{S}(t)$: the complex envelope of FM wave.

أسهل في التفاضل $\quad \tilde{S}(t) = A_c \exp\left(j\, 2\pi k_f \int_0^t m(t)\,dt\right) \longrightarrow ②$

$$\tilde{S}_1(f) = \tilde{H}_1(f) \cdot \tilde{S}(f)$$

F.T. of $\tilde{S}(t)$

o/p = Transfer fn. · I/p

$$= \begin{cases} j\,2\pi a\left(f + \frac{B_T}{2}\right) \cdot \tilde{S}(f) & -\frac{B_T}{2} \leq f \leq \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$\tilde{S}_1(f) = j2\pi f a \cdot \tilde{S}(f) + j\pi a \beta_T \tilde{S}(f)$$

From F.T. Properties "differentiation"

$$\frac{d\,g(t)}{dt} \rightleftharpoons G(f)\,(j2\pi f)$$

$$\therefore \tilde{S}_1(t) = a \cdot \frac{d\tilde{S}(t)}{dt} + j\pi a \cdot \beta_T \cdot \tilde{S}(t)$$

$$\tilde{S}_1(t) = a \cdot \left[\frac{d\tilde{S}(t)}{dt} + j\pi \beta_T \tilde{S}(t)\right]$$

from ② in ①

عوض عن $\tilde{S}(t)$

$$\tilde{S}_1(t) = a \cdot \Big[A_c\,(j2\pi k_f\, m(t))\,\exp\!\left(j2\pi k_f \int m(t)\,dt\right) + j\pi \beta_T \cdot A_c \cdot \exp\!\left(j2\pi k_f \int m(t)\,dt\right)\Big]$$

$$= j\pi a \cdot A_c \cdot \beta_T \left(\frac{2k_f m(t)}{\beta_T} + 1\right) \cdot \exp\!\left(j2\pi k_f \int m(t)\,dt\right)$$

$\therefore$ The desired response of the slope circuit

مركز $H_1(f)$ عند الـ f_c

$$S_1(t) = \mathrm{Re}\left[\tilde{S}_1(t) \cdot \exp(j2\pi f_c t)\right]$$

$$S_1(t) = \mathrm{Re}\left[j\pi a \cdot A_c \beta_T \left(\frac{2k_f m(t)}{\beta_T} + 1\right) \cdot \exp\!\left(j2\pi \left(f_c + k_f \int_0^t m(t)\right) t\right)\right]$$

$$j = 1\angle 90^\circ \quad \therefore \exp\left(\cdots + \frac{\pi}{2}\right)$$

$$= \underbrace{\pi \cdot a A_c \beta_T \left(\frac{2k_f m(t)}{\beta_T} + 1\right)}_{AM} \cos\left(2\pi\left(f_c + \underbrace{k_f \int m(t)\,dt}_{FM}\right) + \frac{\pi}{2}\right)$$

The o/p $S_1(t)$ is a hybrid-modulated wave that has both AM & FM Carrier wave. " the amp. & the Freq. of the Cosine Carrier both vary with $m(t)$".

So, to extract $m(t)$ we will use envelope detector circuit that passes the signal envelope. "ال term الي جنب ال cos"

$$|S_1(t)| = \underbrace{\pi \cdot B_T \cdot a\, A_c}_{\Downarrow} \left(1 + \frac{2k_f}{B_T}\, m(t)\right).$$

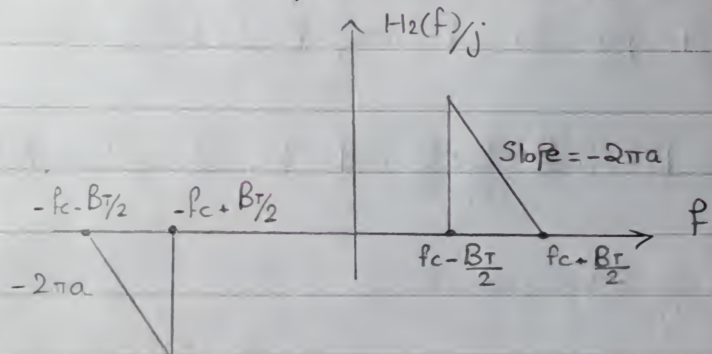
(قيمة dc مجموعة على $m(t)$) Bias term

للحصول على $m(t)$ من غير هذه الكمية

Bias term $= \pi \cdot B_T \cdot a \cdot A_c$

a → Constant that determines the slope of $\frac{H_1(f)}{j}$

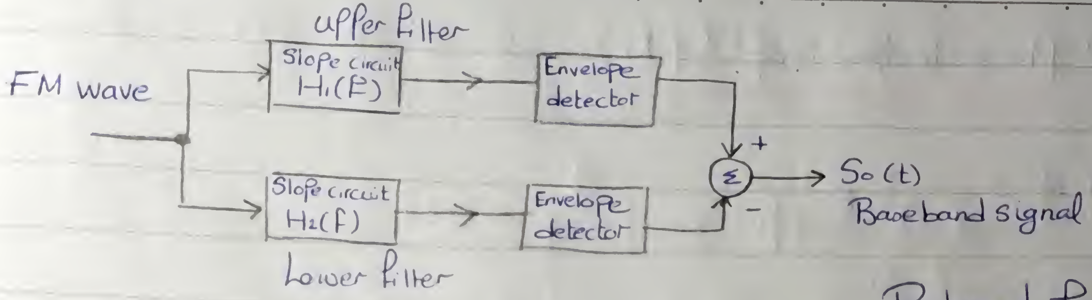
This bias term can be removed by subtracting $S_2(t)$ from $S_1(t)$
$S_2(t)$ has $H_2(f)$ of slope $= -2\pi a$ (has negative a)



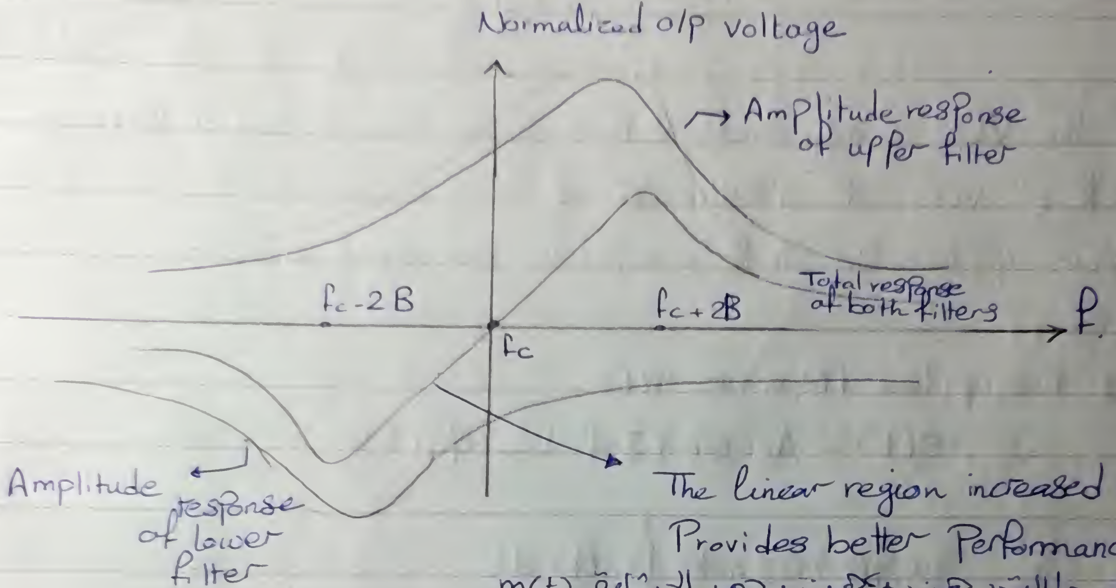
$\therefore\ S_o(t) = |S_1(t)| - |S_2(t)|$

$$|S_2(t)| = \pi B_T a A_c \left[1 - \frac{2k_f}{B_T}\, m(t)\right)$$

$\therefore\ \boxed{S_o(t) = 4\pi k_f a A_c\, m(t)}$ → وهو المطلوب



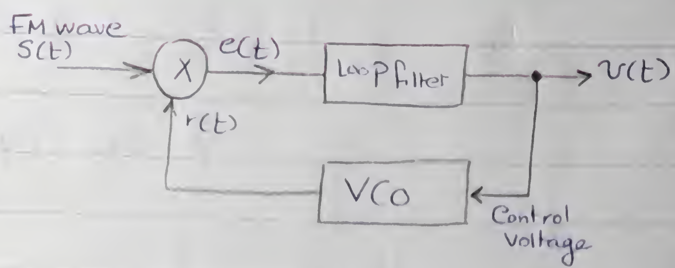
Balanced Freq. discriminator



The linear region increased which Provides better Performance

لأن كل ما التردد هيزيد كل ما هيزيد جهد الاشارة m(t)
بشكل خطي و هذا هو عكس الـ FM في العمل
"demodulation"

② Phase-locked Loop (PLL) :-



Assume that the VCO is adjusted so that when the Control Voltage is zero : ① f of VCO = fc of the carrier

② VCO o/p has 90° shift in respect to the carier wave.

Suppose that the applied FM wave S(t)

$$S(t) = A_c \sin(2\pi f_c t + \phi_1(t))$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(t)\, dt$$

VCO o/p ← $r(t) = A_v \cos(2\pi f_c t + \phi_2(t))$

freq depend on I/p voltage

Same FM eqn $\qquad \phi_2(t) = 2\pi k_v \cdot \int_0^t v(t)\, dt$

that's why

a VCO can be used as

FM modulator

when multiplying $S(t) \times r(t)$

$$K_m \cdot A_c A_v \cdot \sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) \rightarrow \text{High freq. Component}$$

$$+ K_m A_c A_v \sin(\phi_e(t)) \longrightarrow \text{Low-freq. Component}$$

$$e(t) = K_m \cdot A_c \cdot A_v \sin(\phi_e(t))$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right].$$

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$
$$= \phi_1(t) - 2\pi K_v \int_0^t v(t)\,dt$$

$$v(t) = \int_{-\infty}^{\infty} e(\tau)\cdot h(t-\tau)\,d\tau \longrightarrow$$

تناظر ف ال frequency

$V(f) = E(f)\cdot H(f)$ ↓ of Loop filter

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi K_v\cdot v(t)$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi\cdot K_v\cdot \int_{-\infty}^{\infty} e(\tau)\cdot h(t-\tau)\,d\tau$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi \underbrace{K_v\cdot K_m A_c A_v}_{K_o} \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\cdot h(t-\tau)\,d\tau$$

$$= \frac{d\phi_1(t)}{dt} - 2\pi K_o \int_{-\infty}^{\infty} \sin(\phi_e(\tau))\, h(t-\tau)\,d\tau$$

when ($\phi_e = 0$) the locked loop is said to be inphase-lock and the VCO generates its f_c frequency

$$S(t) = A_c\cos(2\pi f_c t + \phi)$$
$$r(t) = A_c \sin(2\pi f_c t + \phi)$$

S, r are 90° phase shifted

$\phi_e = 0$ VCO (f_c)

$\sin[\phi_e(t)] \simeq \phi_e(t)$ for small values

$$\frac{d\phi_e(t)}{dt} + 2\pi K_o \int_{-\infty}^{\infty} \phi_e(\tau)\, h(t-\tau)\,d\tau = \frac{d\phi_1(t)}{dt}$$

⇓ F.T.

$$\phi_E(f)\cdot(j\cancel{2\pi} f) + \cancel{2\pi} K_o\cdot \phi_E(f)\cdot H(f) = \phi_1(f)\cdot(j\cancel{2\pi} f)$$

$$\phi_E(f)\,(jf + K_o H(f)) = \phi_1(f)\cdot f$$

$$\phi_E(f) = \frac{\phi_1(f)\cdot \cancel{f}}{j\cancel{f} + \frac{K_o\cdot H(f)}{f}}$$

$$\phi_e(f) = \frac{1}{1 + L(f)} \cdot \phi_i(f) \quad (3)$$

$$L(f) = k_o \cdot \frac{H(f)}{jf} \quad (4)$$

$$V(f) = \frac{k_o}{k_v} H(f) \cdot \phi_e(f)$$ عوض من (4)

$$V(f) = L(f) \cdot \phi_e(f) \cdot \frac{jf}{k_v}$$ عوض من (3)

$$V(f) = \frac{jf}{k_v} \cdot \frac{L(f)}{1 + L(f)} \phi_i(f)$$

IF $|L(f)| \gg 1$

$$V(f) \simeq \frac{jf}{k_v} \phi_i(f)$$

$$\Big\downarrow \text{ I.F.T.}$$

$$v(t) \simeq \frac{1}{2\pi k_v} \cdot \frac{d\phi_i(t)}{dt}$$

$$\phi_i(t) = 2\pi k_f \int m(t)\, dt \longrightarrow \frac{d\phi_i(t)}{dt} = 2\pi k_f \cdot m(t)$$

$$\therefore \boxed{v(t) \simeq \frac{k_f}{k_v} \cdot m(t)} \rightarrow$$ وهو المطلوب